Portfolio Theory - Exercise Set 6

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Exercises: Preferences

6.1 ([3p])

Assume that $\succ$ is a continuous preference relation on a connected set $\mathcal{X}$, which is endowed with a topology that is first-countable (this allows you to work below with sequences). Let $\mathcal{Z}$ be a dense subset of $\mathcal{X}$. If $U\colon \mathcal{X} \to \mathbb{R}$ is continuous and its restriction to $\mathcal{Z}$ is a numerical representation of $\succ$, then U is also a numerical representation of $\succ$ on all of $\mathcal{X}$. To show this, you verify the following implications:

(a) $x \succ y \Rightarrow U(x) > U(y)$

(b) $U(x) > U(y) \Rightarrow x \succ y$.

Hints:

To show (a) you complete the following steps. Show that there are $z, w \in \mathcal{Z}$ such that $x \succ z \succ w \succ y$. Choose then $z_n, w_n \in \mathcal{Z}$ such that $z_n \to x$ and $w_n \to y$ and finish the proof.

For (b) you show first that $U^{-1}(U(y), \infty) \cap U^{-1}(-\infty, U(x))$ is non-void and select $z, w \in \mathcal{Z}$ such that $U(x) > U(z) > U(w) > U(y)$. Use again convergent sequences.

6.2 ([3p])

Let $\mathcal{X}$ be a connected topological space and $\succ$ a continuous strict preference relation on it.

(a) Let $x \succ y$. Show that $\mathcal{X} = ((y, \rightarrow))^c \cup ((y, x)) \cup ((\leftarrow, x))^c$.

(b) Show that $((y, x))$ is not empty.

(c) Let $\mathcal{Z}$ be dense in $\mathcal{X}$. Show that there are $z, z' \in \mathcal{Z}$ such that $x \succ z \succ z' \succ y$.

6.3 ([3p])

Let $\mathcal{X} = \mathbb{R}^2_+$, endowed with the ordinary topology and with elements $x = (x_1, x_2)$ etc. Define $x \succeq y$ if $x_1 x_2 \geq y_1 y_2$.

(a) Show that $\succeq$ defines a weak preference relation. How does the corresponding strict preference relation look? Is it continuous?

(b) There exists an obvious numerical representation of $\succeq$. Which one? Is this representation continuous?

(c) Give a countable order dense subset.

(d) What are the points in $\mathcal{X}$ that are indifferent of $(1, 1)$? Sketch a few indifference curves.